Problem I

Let $f(x,y) = xy + \dfrac{1}{1-3x+2y}$

a) Compute the partial derivatives f_x' and f_y'

b) Find the gradient of this function at $(x,y) = (0,1)$.

Problem II

Let $z = x^2 + 2x + y^2 + 4y + 1$

a) Find the full differential dz at $(x,y) = (1,2)$

b) Write out the equation for the tangent plane at $(1,2)$.

Problem III

Let $z = z(x,y)$ be the same as in problem II.

a) What are the contour curves (lines) $z = z(x,y) = C$?

b) Find the equation for the tangent line for the contour curve passing through $(x,y) = (1,2)$

Solutions:

Problem I: a) $f_x'(x,y) = y + \frac{3}{(1-3x+2y)^2}$

$f_y'(x,y) = x - \frac{2}{(1-3x+2y)^2}$

b) $(x,y) = (0,1)$

$f_x'(0,1) = 1 + \frac{3}{(1+2)^2} = 4/3$

$f_y'(0,1) = 0 - \frac{2}{(1+2)^2} = -2/9$

$\nabla f|_{(0,1)} = (4/3, -2/9)$

Problem II: a) $f_x' = 2x+2$, $f_y' = 2y+4$

$f_x'(1,2) = 4$, $f_y'(1,2) = 8$

$dz = f_x' dx + f_y' dy$

$dz|_{(1,2)} = 4dx + 8dy$

b) $z_0 = f(x_0, y_0) = f(1,2) = 12$

$x_0 = 1$, $y_0 = 2$

$dz = 4dx + 8dy$ $\quad dz \to z - z_0 = z - 12$

$dx \to x - x_0 = x - 1$

$dy \to y - y_0 = y - 2$

Tangent plane:

$$\boxed{z - 12 = 4(x-1) + 8(y-2)}$$

Problem 3 a) $z(x,y) = C \iff$

$x^2 + 2x + y^2 + 4y + 1 = C$

$(x+1)^2 - 1 + (y+2)^2 - 4 + 1 = C$

$(x+1)^2 + (y+2)^2 = C + 4$

Contour curves = concentric circles centered at $(-1,-2)$.

b) $dz = 4dx + 8dy$ at $(1,2)$

$dz = 0$ on the contour curve

$\Rightarrow 4dx + 8dy = 0$ $\quad dx \to x - x_0 = x - 1$

$dy \to y - y_0 = y - 2$

Tangent: $\boxed{4(x-1) + 8(y-2) = 0}$