Book of Abstracts

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On discrete homoclinic attractors of three-dimensional diffeomorphisms

Abstract: In the theory of dynamical chaos, one of the most important and relevant is its direction that is associated with the study of strange attractors of multidimensional systems (with dimension of phase space ≥ 4 for flows and ≥ 3 for diffeomorphisms). Compared to the lower dimension, there are not too many meaningful results here, but almost all of them are of great importance for the theory of dynamical chaos. In our opinion, one of the most interesting results recently obtained in this direction are connected with the discovery of the so-called discrete homoclinic attractors .

By this term we primarily denote strange attractors of multi-dimensional maps (diffeomorphisms) that contain only one fixed point of saddle type and, hence, they also contain entirely its unstable manifold. In the present talk we give a review on discrete homoclinic attractors of three-dimensional diffeomorphisms both orientable and nonorientable. We discuss the most important peculiarities of these attractors such as their geometric and ho moclinic structures, phenomenological scenarios of their appearance, pseudohyperbolic properties etc. We consider homoclinic attractors of various type such as discrete Lorenz attractors, discrete figure-eight attractors, discrete Shilnikov attractors etc. As illustrative examples, we will consider 3D generalized H´enon maps, 3D maps with the axial symmetry and with the constant Jacobian as well as the model from application – a nonholonomic model of Celtic stone.

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Recent trends in multidimensional chaos theory: pseudohyperbolic, spiral, and hyperchaotic attractors

Abstract: The study of chaotic dynamics in multidimensional systems, its types and nature is one of the most important and fundamental problems in nonlinear science. In this course of lectures we review our recent results in this area paying special attention to the following three topics:

1. Robustly chaotic attractors
2. Spiral chaos
3. Hyperchaos

Robustly chaotic attractors remain chaotic after small perturbations of the system. In [1] it was hypothesized that the robustness of chaoticity is equivalent to the pseudohyperbolicity of the attractor. Pseudo-hyperbolicity is a generalization of hyperbolicity. The main characteristic property of a pseudohyperbolic attractor is that each of its orbits has a positive maximal Lyapunov exponent. In addition, this property must be preserved under small perturbations. In the first lecture we give examples of pseudohyperbolic attractors and explain how to verify pseudo-hyperbolicity with help of quite simple numerical experiments.

According to hypothesis given in [1], if the attractor is not pseudohyperbolic it is a quasiattractor. By Afraimovich and Shilnikov such attractors either contain orbits with vanishing maximal Lyapunov exponent or such orbits appear under small perturbations. The researcher can never be sure whether the chaotic dynamics are observed or it is just a long transient process after which orbits tend to some simple attractor, e.g., stable periodic orbit. Nevertheless, in many cases such attractors are indistinguishable (in a sense) from pseudohyperbolic ones. Moreover, for a wide range of different applications it does not mater whether the attractor is robust or not. In the second lecture we explain the nature of one of the most common types of quasiattractors -- the so-called spiral Shilnikov attractor. We explain how such

attractors can appear. In addition, we provide a specific case when a spiral attractor can be pseudohyperbolic.

One of the recent topics in theory of multidimensional chaos is connected with the study of the so-called hyperchaotic attractors, i.e., such types of attractors whose orbits have at least two positive Lyapunov exponents. Despite the fact that such attractors were discovered long time ago [2], until recently [3], there was no theory explaining the nature of such attractors and how they can appear from simple and strange attractors. In the third lecture we explain mechanisms for the creation of hyperchaotic attractors characterizing by two and three positive Lyapunov exponents.

We will supplement our course of lectures with a description of some numerical methods, toolkits, and software packages that we apply in our studies.

[1] Gonchenko, S., Kazakov, A., & Turaev, D. (2021). Wild pseudohyperbolic attractor in a four-dimensional Lorenz system. Nonlinearity, 34(4), 2018.

[2] Rossler, O. (1979). An equation for hyperchaos. Physics Letters A, 71(2-3), 155-157.

[3] Shykhmamedov, A., Karatetskaia, E., Kazakov, A., & Stankevich, N. (2023). Scenarios for the creation of hyperchaotic attractors in 3D maps. Nonlinearity, 36(7), 3501.

Alexey Ilyin

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Sobolev and spectral inequalities for orthonormal systems in the theory of attractors

Abstract: Sobolev inequalities provide an indispensable tool in the PDE theory. In the theory of attractors upper and lower bounds for orthonormal systems are required for good estimates of the N-traces of the linearized operators on the attractor. We shall discuss in reasonable detail lower bounds of Berezin and Li—Yau-type for the eigenvalues of the Laplace and Stokes operators (including the operators on the sphere) and the corresponding inequalities with correction terms. Upper bounds for the Lp-norms of orthonormal systems systems are called the Lieb—Thirring inequalities and have important applications in mathematical physics, analysis, dynamical systems and attractors, to mention a few. We shall discuss and prove Lieb—Thirring inequalities in the dual form for various boundary conditions including the case of orthonormal divergence free vector functions. We shall also discuss Sobolev inequalities for systems with orthonormal derivatives and their applications to the attractors of certain regularized models in hydrodynamics.

Anna Kostianko

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Inertial Manifolds I: classical theory

Abstract: We revisit the classical theory of the Inertial Manifolds (IMs) including the proof of the main existence theorem based on spectral gap conditions, alternative constructions of IMs via spatial and spatio-temporal averaging methods, method of changing of the dependent variable, etc. The applications will include reaction-diffusion-advection equations, Cahn-Hilliard equations, various regularizations of Navier-Stokes equations, complex Ginzburg-Landau equations. Recent examples of realistic PDEs where the IM does not exist will be also discussed. These lectures will be continued by **Inertial Manifolds II**.

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Persistence of non-hyperbolicity

Abstract: It is believed that non-hyperbolicity is mainly caused by the existence of two kinds of objects: homoclinic tangencies and heterodimensional cycles. A homoclinic tangency is a non-transverse intersection between the stable and unstable manifolds of a hyperbolic periodic point, and a hetero-dimensional process refers to a pair of hyperbolic periodic points with different indices (dimensions of unstable manifolds) whose invariant manifolds intersect cyclically -- due to the dimension defect, one of the intersections must be non-transverse. Remarkably, these seemingly fragile objects have been proven to be persistent, and hence imply the persistence of non-hyperbolicity. In this lecture, we briefly mention some classic results on homoclinic tangencies and focus on some recent progress in the study of heterodimensional cycles, including their persistence in higher regularities and their relationship with homoclinic tangencies and pseudohyperbolic attractors. In the end

we show that the mechanism giving the persistence of heterodimensional cycles also appear near whiskered tori.

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Lection 1

Generation and destruction of multi-frequency quasi-periodic oscillations

Quasi-periodic oscillations represent a wide-spread type of dynamical behavior in many dynamical systems in nature and technology. In very general circumstances, they may be thought of as oscillations composed of two or more components characterized by incommensurate frequencies. Destruction of quasi-periodic oscillations leads to chaos formation. The easiest way to detect chaotic dynamics in numerical experiments is to calculate the largest Lyapunov exponents. Depending on the spectrum of Lyapunov's exponents, various types of oscillations can be classified in flow dynamical system:

- periodic oscillations (0, -, -, …);

- quasi-periodic oscullations (0, 0, …, 0, -, -, …)

- chaotic oscillations (+, 0, -, -, ...)

- hyperchaotic oscillations (+, …, +, 0, -, …-).

Using spectrum of Lyapunov exponents can be classified one more type of dynamical chaos, which has additional zero Lyapunov exponent in the spectrum (+, 0, ..., 0, -, -, ...). This behavior was first described in [1], and the attractor was called the quasi-periodic Hénon attractor, since it is essentially the product of the chaotic Hénon attractor and the ergodic torus.

We consider the simplest models with quasi-periodic dynamics, scenarios of torus destruction. We discuss scenarios which can lead to development of chaos with additional zero Lyapuniv exponents. Consider examples where such kind of attractors occurs [2-3]. Discuss universality of proposed scenario for multi-frequency quasi-periodic oscillations [4].

[1] Broer H. W., Vitolo R., Simó C. Quasi-periodic Hénon-like attractors in the Lorenz-84 climate model with seasonal forcing // EQUADIFF 2003. – 2005. – P. 601-606.

[2] Stankevich N.V., Shchegoleva N.A., Sataev I.R., Kuznetsov A.P. Three-dimensional torus breakdown and chaos with two zero Lyapunov exponents in coupled radio-physical generators Journal of Computational and Nonlinear Dynamics. 2020. Vol. 15, No.11, pp. 111001.

[3] Grines E.A., Kazakov A., Sataev I.R. On the origin of chaotic attractors with two zero Lyapunov exponents in a system of five biharmonically coupled phase oscillators Chaos: An Interdisciplinary Journal of Nonlinear Science 32 No. 9, 2022, 093105.

[4] Kuznetsov A.P., Sedova Yu.V., Stankevich N.V. Coupled systems with quasi-periodic and chaotic dynamics. Chaos, Solitons & Fractals 169 No. 4, 2023, C. 113278.

Lection 2

Hyperchaos associated with Shilnikov discrete attractors in different applications

The scenario of the birth of a hyperchaotic attractor is closely related to the scenario of the emergence of an infinite set of cycles with a multi-dimensional unstable manifold making up its skeleton. For four-dimensional flows there are not so many local bifurcations leading to the birth of a cycle with a two-dimensional unstable manifold: these are the torus bifurcation (often called Neimark-Sacker bifurcation) of the stable cycle, the period-doubling bifurcation of the saddle cycle, and the birth of such a cycle as a result of a saddle-node bifurcation.

The cascade of creation of secondary tori from stable resonance cycles leads to the appearance of the hierarchy of saddle-foci cycles arising as a result of Neimark-Sacker bifurcations [1, 3]. At each stage of this cascade, saddle resonance cycle appear, which undergoes a cascade of period-doubling bifurcations. On the one hand the absorption of these cycles by the attractor ultimately leads to the appearance of hyperchaos. On the other hand this inverse cascade of bifurcations of cycle absorbing is accompanied by the emergence of discrete spiral Shilnikov attractors.

Discrete spiral Shilnikov attractors were discovered in 1986. The term “hyperchaos” was introduced first in 1979. Both events happened almost together but it took more than 30 years to combine them in the one universal scenario [1]. The existence of a large number of examples in which the birth of hyperchaos is associated with the existence of secondary torus birth bifurcations and the occurrence of discrete spiral Shilnikov attractors indicates the prevalence of this scenario. In this talk we discuss examples from different applications (radio-physical generator [2,3], genetic oscillators [4], neuron models [5]), where we can see implementation of this scenario.

[1] Stankevich N.V., Kazakov A.O., Gonchenko S.V. Scenarios of hyperchaos occurrence in 4D Rössler system // CHAOS. 2020. Vol. 30, 123129.

[2] Stankevich N.V., Kuznetsov A.P., Popova E.S., Seleznev E.P. Chaos and hyperchaos via secondary Neimark–Sacker bifurcation in a model of radiophysical generator // Nonlinear Dynamics, 2019, 97, iss. 4, 2355-2370.

[3] Sataev I.R., Stankevich N.V. Cascade of torus birth bifurcations and inverse cascade of Shilnikov attractors merging at the threshold of hyperchaos // CHAOS, 31, 2021, №2, 023140.

[4] Stankevich N., Volkov E. Chaos-Hyperchaos transition in three identical quorum sensing mean field coupled ring oscillators // CHAOS, 31, 2021, 103112.

[5] Stankevich N.V., Bobrovsky A.A., Shchegoleva N.A. Chaos and hyperchaos in two coupled identical Hindmarsh-Rose systems // Regular and chaotic dynamics (submitted).

Dominic Stone

University of Surrey, UK

The non-autonomous Navier-Stokes-Brinkman-Forchheimer equation with Dirichlet boundary conditions: dissipativity, regularity, and attractors

Abstract: We study the 3D Navier-Stokes-Brinkman-Forchheimer equations in a bounded domain endowed with the Dirichlet boundary conditions and non-autonomous external forces. This study includes the questions related with the regularity of weak solutions, their dissipativity in higher energy spaces and the existence of the corresponding uniform attractors.

Dmitry Turaev

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Wildness and Richness of Chaos

Abstract: We show how various global and local bifurcations lead to chaos of ultimate richness: one determined, by a simple computation, the effective dimension

D of the bifurcational problem (an a priori bound on the number of zero Lyapunov exponents) and then shows that all dynamics possible for all systems in the D-dimensional space can emerge at this bifurcation. The techniques can be applied for constructing deep learning algorithms.

Polina Vytnova

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A dynamical viewpoint on resonances of hyperbolic surfaces

Abstract: The Laplacian operator on a geometrically finite hyperbolic surface of infinite area has absolutely continuous spectrum (1/4, +∞) and finitely many eigenvalues in (0,1/4). Its resolvent admits meromorphic extension to complex plane with poles of finite order. The resonances of the surface are the poles of the resolvent. In the case of the surface of constant negative curvature, the resonances turn out to be the poles of a particular analytic function, called the Selberg zeta function. We will discuss this connection and address some questions on distribution and location of the zeros.

Sergey Zelik

ZJNU, China; Univerity of Surrey, UK; HSE Nizhnij Novgorod, Russia;

Keldysh RAN, Moscow, Russia

Inertial Manifolds II

Abstract: These lectures will continue Inertial Manifolds I and will be devoted to the problem about smoothness of Inertial Manifolds (IMs). We will discuss the classical example of G.Sell which shows that IMs may be not even C^2-smooth even in the case where all nonlinearities are smooth and even analytic and present a recent way how this problem may be overcome by using the clever cut-off procedure and the Whitney extension theorem. As the result, under some natural assumptions, we will construct a C^n-smooth IM for every finite n.